SPbU-IP-96-10
hep-ph/9605402

The Extended Chiral Bosonisation And Pion-Diquark Effective Action

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Abstract

We consider bosonisation of the low-energy QCD based on integrating the anomaly of the extended chiral (Eχ) transformation which depends both on pseudoscalar meson and scalar diquark fields as parameters. The relationship between extended chiral and usual chiral anomalies and related anomalous actions is studied. The effective action for the extended chiral field $\mathcal{U}$ depending on complete set of anomalous generators of the Eχ-transformation is given. The terms of this effective action relevant to interaction of pions and $\bar{3}_c$ scalar diquarks are written down explicitly.

PACS: 11.30.Rd, 12.50.-d

1 Introduction

In a conventional understanding of the QCD physics diquarks [1,2] and the quantum anomalies [3] usually are considered as unrelated topics. However in the recent paper [4] a possible connection between these two areas has been proposed. In [4] it has been shown that $J^P = 0^+$ diquarks can be introduced together with pseudoscalar mesons as parameters of certain transformations — extended chiral (Eχ) transformations, including usual chiral as a subgroup. The chiral bosonisation method based on integrating chiral anomaly has been applied in order to obtain the effective action for diquarks and to study their properties. The values for the mass of the ud-diquark $m_{ud} \approx 300MeV$ and mean square charge radius $\langle r^2_{ud}\rangle^{1/2} \approx 0.5fm$ obtained in ref.[4] stimulate an interest for further investigations in this direction.

The present paper is devoted to calculation of the effective interaction of pions and scalar diquarks within the approach based on Eχ-transformations. This is a natural extension of the results of the ref.[4] on the case when the extended chiral field $\mathcal{U}$ depends on pions, scalar diquarks and other fields needed for closure of the Eχ-group [4]. All these states are introduced on a quasi-goldstone manner as angle-like variables corresponding to anomalous (non-preserving measure of quark path integral) generators of Eχ-transformations. An especial status of pions which are as usually supposed to be goldstone particles in the chiral limit is regarded by means of choosing the parameterization of the field $\mathcal{U}$ in a certain form. All results of the pion physics known from the chiral bosonisation approach [5] are reproduced. We do not introduce any new parameters.

The Sect.2 contains main elements of the chiral bosonisation approach adopted to our purposes. At first, we remind the structure of the Eχ-transformations and consider the symmetry properties of the quark path integral. After that, the Eχ-bosonisation is formulated. At last, herein we study the correspondence between the extended chiral and usual chiral anomalies and related anomalous actions.

The Sect.3 is devoted to studying the effective action $W_{eff}(\mathcal{U})$ which contains information about low-energy pion-diquark interaction. We start from the explicit expression for W_{eff} in terms of the field $\mathcal{U}$. The $\bar{3}_c$ scalar diquark sector of W_{eff} is studied and main properties of the diquark are exhibited. Next, we discuss the proper parameterization of the field $\mathcal{U}$ which regards the goldstone nature of pions. We finish the Sect.3 by resulting expression for the effective interaction lagrangian of pions and $\bar{3}_c$ scalar diquarks.

2 The Eχ-bosonisation of the quark path integral

2.1 The Eχ-transformations of the quark fields

Let us remind the formulation of the Eχ-transformations following ref.[4]. Introduce the Majorana-like eight-component spinor Ψ constructed from the ordinary Dirac spinor ψ

$$\Psi = \begin{pmatrix} \psi \\ C\bar{\psi}^T \end{pmatrix}, \qquad \overline{\Psi}^T = \begin{pmatrix} 0 & C^{-1} \\ C & 0 \end{pmatrix} \Psi \tag{1}$$

where C is the charge conjugation matrix. The second relation of (1) is the Majorana condition [7]. The Eχ-transformations of the fermion field with $N = N_c \times N_f$ internal

degrees of freedom, are [4]

$$\delta\Psi = \left[\Phi + \Theta\gamma_5\right]\Psi, \qquad \Phi = i\left(\begin{array}{cc} \alpha & \beta \\ -\beta^* & -\alpha^T \end{array}\right), \quad \Theta = i\left(\begin{array}{cc} \chi & \omega \\ \omega^* & \chi^T \end{array}\right). \tag{2}$$

The matrices $\alpha, \chi, \beta, \omega$ acting in the direct product of colour and flavour spaces are supposed to satisfy the following hermiticity and symmetry properties

$$\alpha = \alpha^+, \quad \chi = \chi^+, \quad \beta = -\beta^T, \quad \omega = \omega^T. \tag{3}$$

The generators ω are of quantum numbers of scalar diquarks. The colourless part of χ corresponds to pions. In ref.[4] it has been shown that Eχ-group is $G = U(2N)$, the Φ generators span the Lie algebra of $H = O(2N)$ and the γ_5 generators Θ belong to the coset $G/H = U(2N)/O(2N)$. The transformations (2) are most general global transformations under whose action the kinetic term of the fermion field is invariant.

To study the quantum theory let us introduce external vector fields v_μ, a_μ, $\phi_{5\mu}$, ϕ_μ generating $\bar\psi\psi$ and $\psi\psi$ vector currents and which have the same matrix structure as the generators (3) respectively. In terms of spinor Ψ the quark lagrangian can be written as

$$\mathcal{L}_\psi = \frac{1}{2}\,\overline{\Psi}\,\hat{G}\,\Psi, \qquad \hat{G} = i\gamma^\mu(\partial_\mu + V_\mu + \gamma_5 A_\mu) \tag{4}$$

where antihermitian fields V_μ and A_μ have the block structure

$$V_\mu = \left(\begin{array}{cc} v_\mu & \phi_{5\mu} \\ \phi^*_{5\mu} & -v^T_\mu \end{array}\right), \qquad A_\mu = \left(\begin{array}{cc} a_\mu & \phi_\mu \\ -\phi^*_\mu & a^T_\mu \end{array}\right). \tag{5}$$

Any local transformation of the fermion field Ψ can be represented equivalently as a transformation of the fields V_μ, A_μ. The finite γ_5-transformation of the fields V_μ, A_μ is $(U_\Theta \equiv \exp\Theta)$

$$\begin{aligned} V_\mu \quad &\rightarrow \quad V^\Theta_\mu = \frac{1}{2}\left\{U^{-1}_\Theta(\partial_\mu + V_\mu + A_\mu)U_\Theta + U_\Theta(\partial_\mu + V_\mu - A_\mu)U^{-1}_\Theta\right\} \\ A_\mu \quad &\rightarrow \quad A^\Theta_\mu = \frac{1}{2}\left\{U^{-1}_\Theta(\partial_\mu + V_\mu + A_\mu)U_\Theta - U_\Theta(\partial_\mu + V_\mu - A_\mu)U^{-1}_\Theta\right\}. \end{aligned} \tag{6}$$

The quark path integral — generating functional for the $\bar\psi\psi$ and $\psi\psi$ $(\bar\psi\bar\psi)$ vector currents reads [9]

$$Z_\psi = \int \mathcal{D}\Psi\ \exp\left\{i\int d^4x\ \mathcal{L}_\psi\right\} = \left[\det\hat{G}\right]^{1/2}. \tag{7}$$

Because of the noninvariance of the measure with respect to γ_5-transformations [10] the quark path integral (7) does not invariant under action of transformations (2) or (6). Thus Z_ψ has the structure $Z_\psi = \exp i(W_{an} + W_{inv})$ where W_{an} and W_{inv} are noninvariant and invariant functionals of the external fields correspondingly. This means that there exists an Eχ-anomaly, which is defined as follows

$$\mathcal{A}^a = \frac{1}{i}\frac{\delta\ln Z_\psi}{\delta\Theta_a}\bigg|_{\Theta_a=0} = \frac{\delta W_{an}}{\delta\Theta_a}\bigg|_{\Theta_a=0} \tag{8}$$

where $\{\Theta\} = \{\chi, \omega, \omega^*\}$ is the set of parameters of anomalous transformations; $\Theta = \Theta^a T^a$ and T^a are (block) antihermitian generators.

2.2 The Eχ-bosonisation

In refs.[6] it has been shown that if the quark path integral has an anomaly with respect to some transformations then we can perform a bosonisation of the variables generating these anomalous transformations. This general result plays an important role in our suggestion to consider scalar diquarks as associated with ω while pseudoscalar mesons as usually will be associated with χ.

Let us now formulate the Eχ-bosonisation. This can be done by generalization of the chiral bosonisation approach developed in refs.[5,6].

An important ingredient of the chiral bosonisation of the low-energy QCD is the hypothesis that there exists some low-energy region L where the noninvariant or anomalous fluctuations of quark variables are essential while out of this region they are negligible. The region L can be formulated as a some region of the spectra the Dirac operator $\hat{G}$, as it done below. Because Z_ψ is the determinant, $Z_\psi = \det^{1/2}\hat{G} \equiv \prod \lambda^{1/2}$, $\lambda \in \mathrm{spec}\,\hat{G}$, the hypothesis of existing the low-energy region L means that $Z_\psi = Z_\psi^L\, \widetilde{Z}_{inv}$. The functional $Z_\psi^L = \prod_{\lambda \in L} \lambda^{1/2}$ contains all information on Θ-noninvariant processes while $\widetilde{Z}_{inv}$ is invariant functional. Thus all contributions in Eχ-anomaly comes from Z_ψ^L.

The Eχ-bosonisation can be formulated as follows

$$\begin{aligned} Z_\psi^L(V,A) &= Z_{inv} \int \mathcal{D}\mu(\Theta) \left[\frac{Z_\psi^L(V,A)}{Z_\psi^L(V^\Theta, A^\Theta)} \right] \\ &= Z_{inv} \int \mathcal{D}\mu(\Theta) \exp\left\{ i W_{eff}(\Theta) \right\} \end{aligned} \tag{9}$$

where $\mathcal{D}\mu(\Theta)$ is an invariant measure on the corresponding coset space G/H and Z_{inv} is some invariant functional. The effective action $W_{eff}(\Theta)$ is defined as

$$W_{eff}(\Theta) = W_{an}(V,A) - W_{an}(V^\Theta, A^\Theta) = -\int d^4x \int_0^1 ds\, \mathcal{A}^a(V^{s\Theta}, A^{s\Theta})\Theta^a. \tag{10}$$

where argument $s\Theta$ means corresponding transformation of background fields. The effective action W_{eff} is the main object which we are looking for. It describes the effective low-energy interaction of the Θ-variables. The explicit expression for W_{eff} is written in the next Section, where it is studied.

2.3 The Eχ-anomaly and W_{an}

In (10) the action W_{eff} is defined through Eχ-anomaly or, equivalently, through W_{an}. Thus we have to find the explicit expressions for $\mathcal{A}^a$ or W_{an} in terms of the external fields.

To find the Eχ-anomaly and the related anomaly action one should regularize the functional Z_ψ. Usually the chiral bosonisation method is armed by finite-mode regularization scheme [11, 5] which consists in continuation to Euclidian domain and specifying the low-energy region L as some region of spectra of the hermitian operator $\hat{G}_E$. The functional Z_ψ^L in Euclidian space is

$$Z_{\psi,E}^L = \det{}^{1/2} \left\{ \hat{G}_E\, \theta\Big(1 - \frac{(\hat{G}_E - M)^2}{\Lambda^2}\Big) \right\}, \quad \theta(A) = \int\limits_{-\infty}^{\infty} \frac{ds\, \exp(isA)}{2\pi i (s - i0)}. \tag{11}$$

where Λ, M are parameters defining the region L: $\lambda \in L$ if $-\Lambda + M \leq \lambda \leq \Lambda + M$; λ is an eigenvalue of $\hat{G}_E$.

The straightforward calculations based on the regularized functional (11) lead us to the following simple but useful result.

In fact, the expression for the Eχ-anomaly $\mathcal{A}^a$ can be obtained from the usual chiral's one, expressed in terms of v_μ, a_μ only. Indeed, in our case the operator $\hat{G}$ is presented in the form of the usual Dirac operator (see (4)) where instead of v_μ, a_μ we have block antihermitian external fields V_μ and A_μ given by (5). Because V_μ, A_μ are transformed infinitesimally by Θ like v_μ, a_μ are transformed by usual chiral χ, the $E\chi$-anomaly can be obtained from the known chiral one [3,5,11] by means of formal substitution

$$v_\mu \to V_\mu, \quad a_\mu \to A_\mu, \quad t^a \to T^a, \tag{12}$$

accomplished by taking additionally trace in two-dimensional "block" space and by taking into account the overall factor 1/2 coming from the power of determinant in (11). The same arguments are valid when $\hat{G}$ depends on external scalar fields as well.

As it is well known [3,5], the anomaly consists out of topologically trivial $\mathcal{A}^+$ and non-trivial $\mathcal{A}^-$ parts: $\mathcal{A} = \mathcal{A}^+ + \mathcal{A}^-$. The part $\mathcal{A}^-$ after integration in (10) form the Wess-Zumino-Witten action responsible for $\mathcal{P}$-odd processes. The dynamics (decay constants, masses ets.) is contained in that part of W_{eff} which is related to $\mathcal{A}^+$ and this part is relevant for our investigation. Instead of $\mathcal{A}^+$ we present here the corresponding part of W_{an}

$$\begin{aligned} W^+_{an} &= \int d^4x \Big\{ \frac{\Lambda^2 - M^2}{8\pi^2} \operatorname*{tr}_{(b,c,f)} A^2_\mu \\ &- \frac{1}{96\pi^2} \operatorname*{tr}_{(b,c,f)} \left(F^2_{\mu\nu} - [A_\mu, A_\nu]^2 - 2F_{\mu\nu}[A_\mu, A_\nu] - 2[D_\mu, A_\mu]^2 + 4(A^2_\mu)^2 \right) \Big\}. \end{aligned} \tag{13}$$

Herein $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu]$ and $D_\mu = \partial_\mu + V_\mu$.

Note that the arguments presented above about substitution of the fields are valid for the expression of W_{an} as well. Therefore the results of ref.[5] can be used straightforwardly in order to obtain the expressions both for W_{an} and $\mathcal{A}^a$ if it is needed to take into account the scalar fields together with the vector ones.

3 Pion-diquark effective action

3.1 An expression for the W_{eff}

Due to relationship between Eχ-anomaly and chiral anomaly established in previous Section, the pion-diquark effective action in terms of Eχ-field $\mathcal{U}$ looks like as pure pion one [5] and reads

$$\begin{aligned} W_{eff} &= \int d^4x \Big\{ \frac{f^2_\pi}{48} \operatorname*{tr}_{(b,c,f)} (D_\mu \mathcal{U})(D^\mu \mathcal{U}^+) + \frac{1}{192\pi^2} \operatorname*{tr}_{(b,c,f)} \Big((D^2_\mu \mathcal{U})(D^2_\nu \mathcal{U}^+) + \\ &+ \frac{1}{2} (D_\mu \mathcal{U})(D_\nu \mathcal{U}^+)(D^\mu \mathcal{U})(D^\nu \mathcal{U}^+) - \left[(D_\mu \mathcal{U})(D^\mu \mathcal{U}^+) \right]^2 + \\ &+ 2(D_\mu F^{\mu\nu}) \left[(D_\nu \mathcal{U}) \mathcal{U}^+ + (D_\nu \mathcal{U}^+) \mathcal{U} \right] - \frac{1}{2} [F_{\mu\nu}, \mathcal{U}][F^{\mu\nu}, \mathcal{U}^+] \Big). \Big\} \end{aligned} \tag{14}$$

Herein we drop the Wess-Zumino-Witten action. For sake of simplicity we also drop all external fields except the vector field v_μ containing relevant dynamical fields of gluons $v_\mu = G_\mu = gG^a_\mu(\frac{\lambda^a}{2i})$. From now on V_μ and its strength tensor $F_{\mu\nu}$ are block-diagonal matrices

$$V_\mu = \left(\begin{array}{cc} G_\mu & 0 \\ 0 & -G^T_\mu \end{array} \right), \quad F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu] = \left(\begin{array}{cc} G_{\mu\nu} & 0 \\ 0 & -G^T_{\mu\nu} \end{array} \right). \tag{15}$$

The covariant derivative acts as $(D_\mu *) = (\partial_\mu *) + [V_\mu, *]$ with V_μ given by (15).

The parameters Λ and M are related to pion decay constant $f_\pi = 132MeV$

$$f^2_\pi = \frac{3}{2\pi^2}(\Lambda^2 - M^2). \tag{16}$$

This relation has been used in (14) to express the only parameter of W_{eff} in a model independent way.

The Eχ-field $\mathcal{U}$ in (14) is defined as $\mathcal{U} = U^2_\Theta$ where $U_\Theta = \exp\Theta$ as above. But below we *redefine* $\mathcal{U}$. Before explanation of the reasons of redefinition $\mathcal{U}$ we would like to consider the diquark sector of the effective action (14).

3.2 The $\bar{3}_c$ scalar diquark sector of W_{eff}

The $\bar{3}_c$ scalar diquark sector of W_{eff} corresponds to the case when Θ contains $\bar{3}_c$ scalar diquark fields *only*, i.e.

$$\Theta = i \left(\begin{array}{cc} 0 & \omega \\ \omega^* & 0 \end{array} \right), \quad (\omega)^{ab}_{jk} = \frac{1}{f_\omega}\,\omega_c\,(i\sigma_2)_{jk}\,\epsilon^{cab}, \quad (\omega^*)^{ab}_{jk} = \frac{1}{f_\omega}\,\omega^*_c\,(i\sigma_2)_{jk}\,\epsilon^{cab} \tag{17}$$

where j, k are flavour and a, b, c are colour indices. f_ω is the diquark decay constant which will be determined below.

The reduction (17) has been considered in [4]. It has been shown that in this case $G = SU(4) \sim O(6)$, $H = SU(3) \times U(1)$ and scalar $\bar{3}_c$ ud-diquarks ω belong to $G/H = CP^3 = SU(4)/SU(3) \times U(1)$.

The diquark mass term comes from the last term of (14), because the later contains the term $G^a_{\mu\nu}G^{\mu\nu,a}\omega_c\omega^*_c$. Substituting quasi-classically instead of $G^a_{\mu\nu}G^{\mu\nu,a}$ its vacuum expectation value we obtain the following expression for the inverse diquark propagator

$$[\mathcal{D}_\omega(p^2)]^{-1} = \frac{2f^2_\pi}{3f^2_\omega}p^2 + \frac{1}{12\pi^2 f^2_\omega}p^4 - \frac{C_g}{36f^2_\omega} \tag{18}$$

where C_g is the gluon condensate, $C_g =< \frac{g^2}{4\pi^2}(G^c_{\mu\nu})^2 >$. The second term in (18) comes from the "tachionic" term in (14). From (18) we see that the mass of ud-diquark ω is defined by the gluon condensate

$$M^2_\omega = 2\pi^2 f^2_\pi \left(\sqrt{1 + \frac{C_g}{12\pi^2 f^4_\pi}} - 1 \right). \tag{19}$$

For actual value of gluon condensate $C_g = (365MeV)^4$ we get $M_\omega \approx 300MeV$. As it was mentioned in ref.[4] the correction of this evaluation due to quark masses is provided by

$M^2_\omega(m_q \neq 0) = M^2_\omega(m_q = 0) + m^2_\pi$. This gives $M^2_\omega(m_q \neq 0) \approx 340MeV$, which falls into the region allowed in the other models [2]. The low mass of diquark can be explained by a reason that this is in fact "current" mass because we do not take into account the interaction with gluons which leads to "dressing" the diquark.

The diquark decay constant f_ω is defined by requirement that the residue of the diquark propagator at $p^2 = M^2_\omega$ is unity,

$$f^2_\omega = \frac{2}{3} f^2_\pi + \frac{1}{6\pi^2} M^2_\omega = \frac{2}{3}\sqrt{f^4_\pi + \frac{C_g}{12\pi^4}}. \tag{20}$$

For the values $C_g = (365MeV)^4$ and $f_\pi = 132MeV$ this gives $f_\omega \approx 120MeV$. The additional term depended upon the diquark mass in (20) comes due to tachion pole of the propagator (18) and is about 5% respect to the first term therein. In rough estimations it can be neglected.

In fact, the relation $f_\omega = \sqrt{\frac{2}{3}} f_\pi$ reproduce the results of the ref.[12] where instead of $f_{\pi,\omega}$ therein $g_{\pi,\omega}$ has been used[1]. In the chiral limit which we consider here, $f_{\pi,\omega}$ and $g_{\pi,\omega}$ are related through quark condensate $C_q (< 0)$ as

$$g_\pi = -\frac{C_q}{2f_\pi}, \quad g_\omega = -\sqrt{\frac{2}{3}}\frac{C_q}{2f_\omega}. \tag{21}$$

From $f_\omega = \sqrt{\frac{2}{3}} f_\pi$ follows that $g_\pi = g_\omega$ what is the result of ref.[12] obtained by QCD sum rules technique.

3.3 The Eχ-field $\mathcal{U}$ with all possible Θ-variables

Consider now the most general case when Eχ-field $\mathcal{U}$ depends on all possible Θ-variables. We take Θ in the general form (2)

$$\Theta = i \begin{pmatrix} \chi & \omega \\ \omega^* & \chi^T \end{pmatrix}. \tag{22}$$

The $J^P = 0^-$ meson states are contained in χ and $J^P = 0^+$ diquark states are contained in ω

$$\chi = \frac{\pi}{f_\pi} + \frac{\chi_8}{f_8}, \quad \omega = \frac{\omega_{\bar{3}}}{f_{\bar{3}}} + \frac{\omega_6}{f_6}. \tag{23}$$

The $SU(3)$ colour group representations of the fields are shown in notations explicitly, except for the colour singlet fields describing pions

$$\pi = \pi^i \left(\frac{\sigma^i_f}{\sqrt{2}} \otimes 1_c\right) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix}_f \otimes 1_c, \tag{24}$$

where σ^i_f are isospin Pauli matrices and π^i, $i = 1, 2, 3$ are elements of the isotriplet, related to $\pi^{\pm,0}$ via (24). Due to exceptional role of pions it is natural to extract them from the other states in Θ. Denote

$$\Pi = i \begin{pmatrix} \frac{\pi}{f_\pi} & 0 \\ 0 & \frac{\pi^T}{f_\pi} \end{pmatrix}, \quad \Omega = i \begin{pmatrix} \frac{\chi_8}{f_8} & \frac{\omega_{\bar{3}}}{f_{\bar{3}}} + \frac{\omega_6}{f_6} \\ \frac{\omega^*_{\bar{3}}}{f_{\bar{3}}} + \frac{\omega^*_6}{f_6} & \frac{\chi^T_8}{f_8}. \end{pmatrix} \tag{25}$$

[1] In [12] g_ω is denoted by g^D_3

Then the sum $\Theta = \Pi + \Omega$ represents decomposition on the colour singlet (Π) and non-singlet (Ω) parts.

Now we are going to discuss the reasons for redefinition of the $\mathcal{U}$ if it is taken in the usual form $\mathcal{U} = \exp 2\Theta$. Also we describe the most obvious method of its redefinition which means changing variables from unphysical to physical ones.

On this way we take in advance the *goldstone* nature of pions which are related to exact $SU(2)_L \times SU(2)_R$ symmetry of the massless QCD lagrangian. The goldstone particles have an important and general property: they can interact between themselves and with any other particle only through vertices with derivatives. As far as pions are considered as goldstone particles this property must be preserved in description of those physical processes in whose pions take part [8].

However this property will be lost if one identifies $\mathcal{U}$ in a usual way as $\mathcal{U} = \exp 2\Theta = \exp 2(\Pi + \Omega)$ where both Π and Ω are put to be nonzero. The puzzle arises due to the term $\mathrm{tr}[F_{\mu\nu},\mathcal{U}][F^{\mu\nu},\mathcal{U}^+]$ in W_{eff} which contains pion interaction terms starting from the Θ^4. This happens only because of specific dependence on pion fields in $\mathcal{U} = \exp 2(\Pi + \Omega)$ with Π and Ω are given by (25). Because $[\Pi, F_{\mu\nu}] = 0$, there are no pure pion vertices (together with $G_{\mu\nu}G_{\mu\nu}$) in $\mathrm{tr}[F_{\mu\nu},\mathcal{U}][F^{\mu\nu},\mathcal{U}^+]$. But because $[\Pi, \Omega] \neq 0$, we find that the term $\mathrm{tr}[F_{\mu\nu},\mathcal{U}][F^{\mu\nu},\mathcal{U}^+]$ contains π–$\omega_{\bar{3}}, \omega_6, \chi_8$ interaction terms without derivatives. All this indicates that $\mathcal{U}$ taken in the form does not preserve the goldstone nature of pions in the π–Ω interaction sector of W_{eff}.

Therefore when the term $\mathrm{tr}[F_{\mu\nu},\mathcal{U}][F^{\mu\nu},\mathcal{U}^+]$ (and only this term in W_{eff}) contains non-goldstone type of pion vertices, we have to redefine the field $\mathcal{U}$ in order to eliminate the dependence of $\mathrm{tr}[F_{\mu\nu},\mathcal{U}][F^{\mu\nu},\mathcal{U}^+]$ on pion fields. This means that instead of $\mathcal{U} = \exp 2(\Pi + \Omega)$ we have to take some $\mathcal{U}_{phys} = \mathcal{U}_{phys}(\Pi, \Omega)$ which must satisfy the following condition

$$\frac{\delta}{\delta \pi^i}\, \mathrm{tr}[F_{\mu\nu},\mathcal{U}_{phys}][F^{\mu\nu},\mathcal{U}^+_{phys}] = 0 \tag{26}$$

provided $\mathcal{U}|_{\Omega=0} = \mathcal{U}_{phys}|_{\Omega=0}$ keeps the pion sector of W_{eff} unchanged.

To determine $\mathcal{U}_{phys}$ we can use the relationship between the finite γ_5-transformation and the corresponding Eχ-field. Choosing any specific form of parameterization of the finite γ_5-transformation we get some specific form of the Eχ-field trying to satisfy the condition (26). The following parameterization of the element $g \in G/H$ is suitable for our purposes

$$g = \exp \Pi\gamma_5 \, \exp \Omega\gamma_5 = U_\Pi U_\Omega \left(\frac{1+\gamma_5}{2}\right) + U_\Pi^{-1} U_\Omega^{-1} \left(\frac{1-\gamma_5}{2}\right) \tag{27}$$

where $U_\Pi = \exp \Pi$, $U_\Omega = \exp \Omega$. The corresponding Eχ-field is $\mathcal{U}_{phys}$

$$\mathcal{U}_{phys} = U_\Pi \, U_\Omega \, (U_\Pi^{-1} \, U_\Omega^{-1})^{-1} = U_\Pi \, U_\Omega^2 \, U_\Pi \tag{28}$$

because due to $[\Pi, F_{\mu\nu}] = 0$ from (28) immediately follows

$$\mathop{\mathrm{tr}}_{(b,c,f)}[F_{\mu\nu},\mathcal{U}_{phys}][F^{\mu\nu},\mathcal{U}^+_{phys}] = \mathop{\mathrm{tr}}_{(b,c,f)}[F_{\mu\nu}, U_\Omega^2][F^{\mu\nu}, U_\Omega^{-2}] \tag{29}$$

and therefore the condition (26) is satisfied.

Note that reparameterization of the Eχ-field does not affect the properties of the states (23) entering in the Θ. It only corrects the structure of the π–Ω interaction

and does not touch any term from the Ω-sector of the W_{eff}. This occurs because the reparameterization of the element $g \in G/H$ does not change the W_{eff} considered as a function of the field $\mathcal{U}$. The functional dependence of W_{eff} on $\mathcal{U}$ is closely determined by the anomaly action W_{an} whose expression in terms of external fields (13) obviously is independent on parameterization of the finite γ_5-transformation.

The described scheme can be easily extended on the case of three flavours. The presence of the additional terms due to nonzero quark masses does not influence the arguments just explained. The condition (26) is the only condition which has to be satisfied in order to guarantee that in the chiral limit the pseudoscalar colourless mesons appear to be exact goldstones.

3.4 Pion-diquark effective interaction

In what follows we will assume that $\mathcal{U}$ in W_{eff} always is $\mathcal{U}_{phys}$ given by (28). Introduce the following notations

$$\mathcal{U}_\Omega = U_\Omega^2, \quad \Xi = U_\Pi = \left(\begin{array}{cc} \xi & 0 \\ 0 & \xi^T \end{array} \right) \tag{30}$$

where as conventionally $\xi = \exp(i\pi/f_\pi)$ and the pion matrix π is given by (24). In these notations Eχ-field $\mathcal{U}$ acquires the form

$$\mathcal{U} = \Xi \mathcal{U}_\Omega \Xi. \tag{31}$$

Substituting $\mathcal{U}$ given by (31) into (14) we obtain the following expression for π–Ω interaction lagrangian

$$\begin{aligned} L_{\pi,\Omega} &= \frac{f_\pi^2}{48} \mathop{\mathrm{tr}}_{(b,c,f)} \Big\{ [\Xi^+(\partial_\mu \Xi), \mathcal{U}_\Omega][\Xi(\partial_\mu \Xi^+), \mathcal{U}_\Omega^+] \\ &\quad + 2\mathcal{U}_\Omega^+ (D_\mu \mathcal{U}_\Omega)\, \Xi(\partial_\mu \Xi^+) + 2\mathcal{U}_\Omega (D_\mu \mathcal{U}_\Omega^+)\, \Xi^+(\partial_\mu \Xi) \Big\} + L_{\pi,\Omega}^{(4)} \end{aligned} \tag{32}$$

where $L_{\pi,\Omega}^{(4)}$ contains vertices of order p^4. The expression for $L_{\pi,\Omega}^{(4)}$ is rather long and it will be presented elsewhere. The terms of (32) shown explicitly determine the main contributions in the vertex Green functions at small external momenta. Here we would like to stress again that the term $\mathrm{tr}[F_{\mu\nu}, \mathcal{U}][F^{\mu\nu}, \mathcal{U}^+]$ of W_{eff} does not contribute in $L_{\pi,\Omega}$ (see Sect.3.3). This term is closely contained in Ω-sector of W_{eff} which can be found if we put $\mathcal{U} = \mathcal{U}_\Omega$ in (14).

The interaction of pions and $\bar{3}$ scalar diquarks can be obtained from (32). Using (17) we find

$$L_{\pi,\bar{3}} = -\frac{f_\pi^2}{12} \sin^2 |\omega_{\bar{3}}| \mathop{\mathrm{tr}}_{(f)} \Big\{ \partial_\mu u^+ \partial_\mu u \Big\} + L_{\pi,\bar{3}}^{(4)} \tag{33}$$

where

$$u = \xi^2 = \exp(2i\pi/f_\pi), \quad |\omega_{\bar{3}}| \equiv 2\sqrt{\omega_{\bar{3}}^a \omega_{\bar{3}}^{*a}}/f_\omega. \tag{34}$$

The expression (33) provides us an information about pion-diquark effective interaction at low energies. Using the vertices contained in (33) one can study pion-nucleon scattering at small pion momenta provided we have a good model description of bounding diquark together with the third quark into nucleon. Of cause, in order to obtain

the predictions comparable with the data the influence of quark masses should be taken into account. However it is important that we succeeded in description of pion-diquark interaction from universal point of view of extended chiral transformation approach.

Acknowledgments

The author is very indebted to Prof. Yu.V. Novozhilov and Dr. D.V. Vassilevich for many interesting and stimulating discussions.

References

[1] M. Gell-Mann, Phys. Lett. 8 (1964) 214;
M. Ida, R. Kobayashi, Progr. Theor. Phys. 36 (1966) 846;
D.B. Lichtenberg, L.J. Tassie, Phys. Rev. 155 (1967) 1601.

[2] M. Szczekowski, Int. J. Mod. Phys. A 4, No. 16 (1989) 3985;
M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson, D.B. Lichtenberg, Rev. Mod. Phys. 65 (1993) 1199.

[3] S.L. Adler, Phys. Rev. 177 (1969) 2426;
J.B. Bell, R. Jackiw, Nuovo Cimento 60 (1969) 147;
W.A. Bardeen, Phys. Rev. 184 (1969) 147.

[4] Yu.V. Novozhilov, A.G. Pronko, D.V. Vassilevich, Phys. Lett. B 343 (1995) 358; Phys. Lett. B 351 (1995) 601.

[5] A.A. Andrianov, V.A. Andrianov, V.Yu. Novozhilov, Yu.V. Novozhilov, Lett. Math. Phys. 11 (1986) 217; Theor. Math. Phys. 67 (1986) 198; JETP Lett. 42 (1986) 579; Phys. Lett. B 186 (1987) 401; Theor. Math. Phys. 70 (1987) 1; Phys. Lett. B 203 (1988) 160;
A.A. Andrianov, Yu.V. Novozhilov, Phys. Lett. B 153 (1985) 422; A.A. Andrianov, Phys. Lett. B 157 (1985) 425.

[6] Andrianov A.A., Novozhilov Yu.V., Theor. Math. Phys. 69 No.1 (1986) 78; Phys. Lett. B 181 (1986) 129.

[7] R. Ball, Phys. Rep. 182 (1989) 1.

[8] S.L. Adler, Phys. Rev. 139 (1965) 1638;

[9] Yu.V. Novozhilov, A.G. Pronko and D.V. Vassilevich, Phys. Lett. B 321 (1994) 425; Theor. Math. Phys. 101 No.2 (1994) 211.

[10] K. Fujikawa, Phys. Rev. D 21 (1980) 2848.

[11] A.A. Andrianov and L. Bonora, Nucl. Phys. B 233 (1984) 232.

[12] H.G. Dosch, M. Jamin, B. Stech, Z. Phys. C 42 (1989) 167.